**Math III Unit 3 Part 1: QUADRATIC MODELING AND EQUATIONS  
Lauren Winstead, Heritage High School**

**Main topics of instruction:**

1) The Real Number System

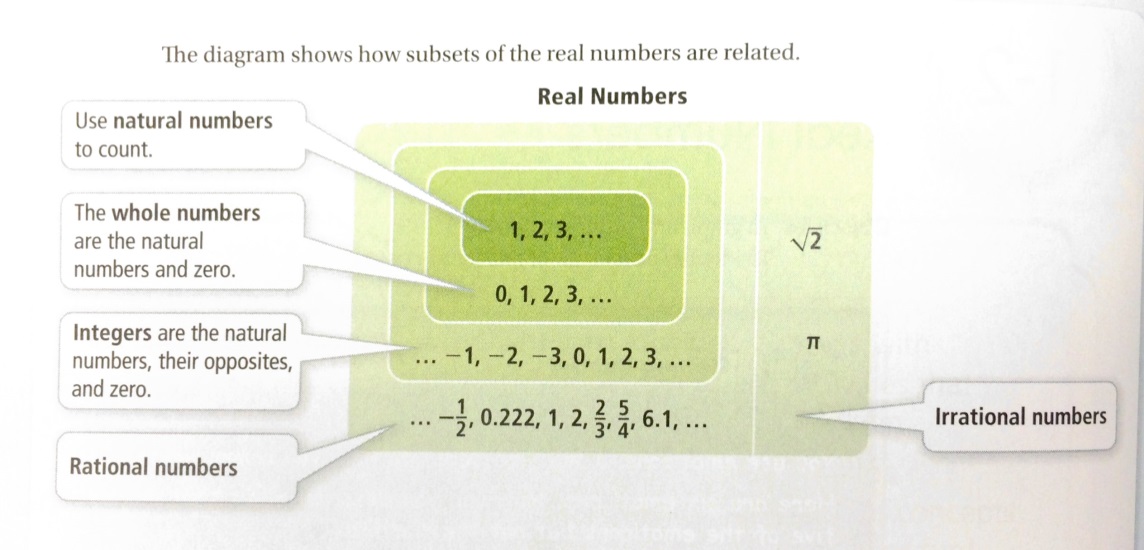
2) Factoring and solving quadratic equations

3) Graphing quadratic equations

4) Complex Numbers

**Day 1: The Real Number System and Factoring**

There are two types of real numbers: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ numbers and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ numbers. Every real number can be graphed as a point on the number line.



Rational Numbers Irrational Numbers

Classify the following as rational or irrational. If a number is rational, state if it is a natural number, whole number, integer, or simply rational.

1. 4 b) -3 c) d) 0.125

e) f) g) 0 f) π

**Critical Thinking:** In each scenario, answer Always, Sometimes, or Never. If the answer is Sometimes, give examples of each outcome.

1. The sum of a rational number and a rational number is a rational number.
2. The product of two rational numbers is a rational number.
3. The sum of a rational number and an irrational number is an irrational number.
4. The product of a rational number and an irrational number is an irrational number.
5. The sum of two irrational numbers is an irrational number.
6. The product of two irrational numbers is an irrational number.

**Factoring – Quadratics**

**Greatest Common Factor (GCF):** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**Example 1: You try!**

Factor and solve Factor and solve

**Standard form of a quadratic expression:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example 2:** Factor and solve

**You try!** Factor and solve the following.

1. b) c)

**Quick!** Throw these in a calculator! What do you notice about where the parabolas cross the x-axis?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

These are called **zeros!** They are also called \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**Factoring – Polynomials**

**Example 1: Factoring Using the GCF You try!**

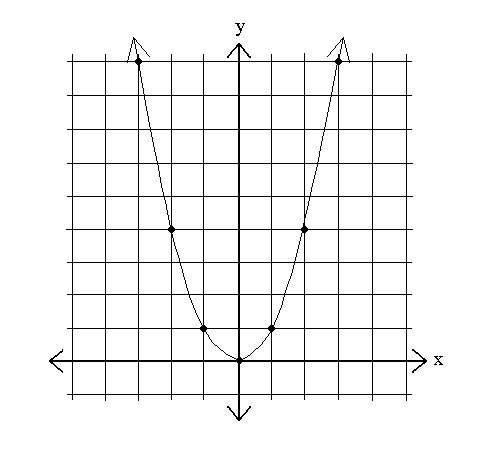
Factor and solve Factor and solve

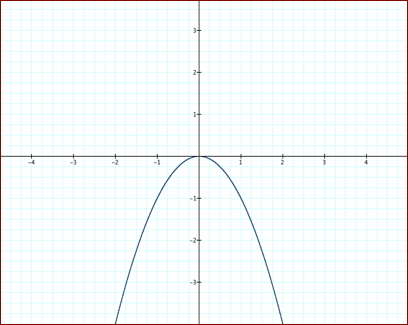
**Example 2: Factoring Using Grouping You try!**

Factor and solve Factor and solve

**Day 2: Finding the Equation of a Parabola in Standard Form**

The graph of a quadratic function is called a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.





**Standard Form of a Quadratic Function:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Axis of Symmetry:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Can be found with the formula:

**Vertex:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

How can I find the y value of the vertex? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



**Example 1:** Find the vertex and axis of symmetry,

then graph

**You try!** Find the vertex and axis of symmetry, then graph .



**Finding a Quadratic Equation in Standard Form**

**Example 2:** A parabola has three points: (2, 3), (3, 13), and (4, 29). Find a quadratic equation (model) in standard form that will fit the parabola.

**You try!** A parabola has three points: (1, 0), (2, -3), and (3, -10). Find a quadratic equation (model) in standard form that will fit the parabola.

**Example 3:** Anthony throws a football across the field while standing on top of the bleachers. The data that follows gives the height of the ball in feet versus the seconds since the ball was thrown.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| time | .2 | .6 | 1 | 1.2 | 1.5 | 2 | 2.5 | 2.8 | 3.4 | 3.8 | 4.5 |
| height | 92 | 110 | 130 | 134 | 142 | 144 | 140 | 132 | 112 | 90 | 44 |

Write a quadratic model for this data. (Round to two decimal places.)

**Day 3: Vertex Form and Translating Parabolas**

Standard Form of a Parabola: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Vertex Form of a Parabola:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ where the vertex is ( , ).

**Using Vertex Form to Graph**



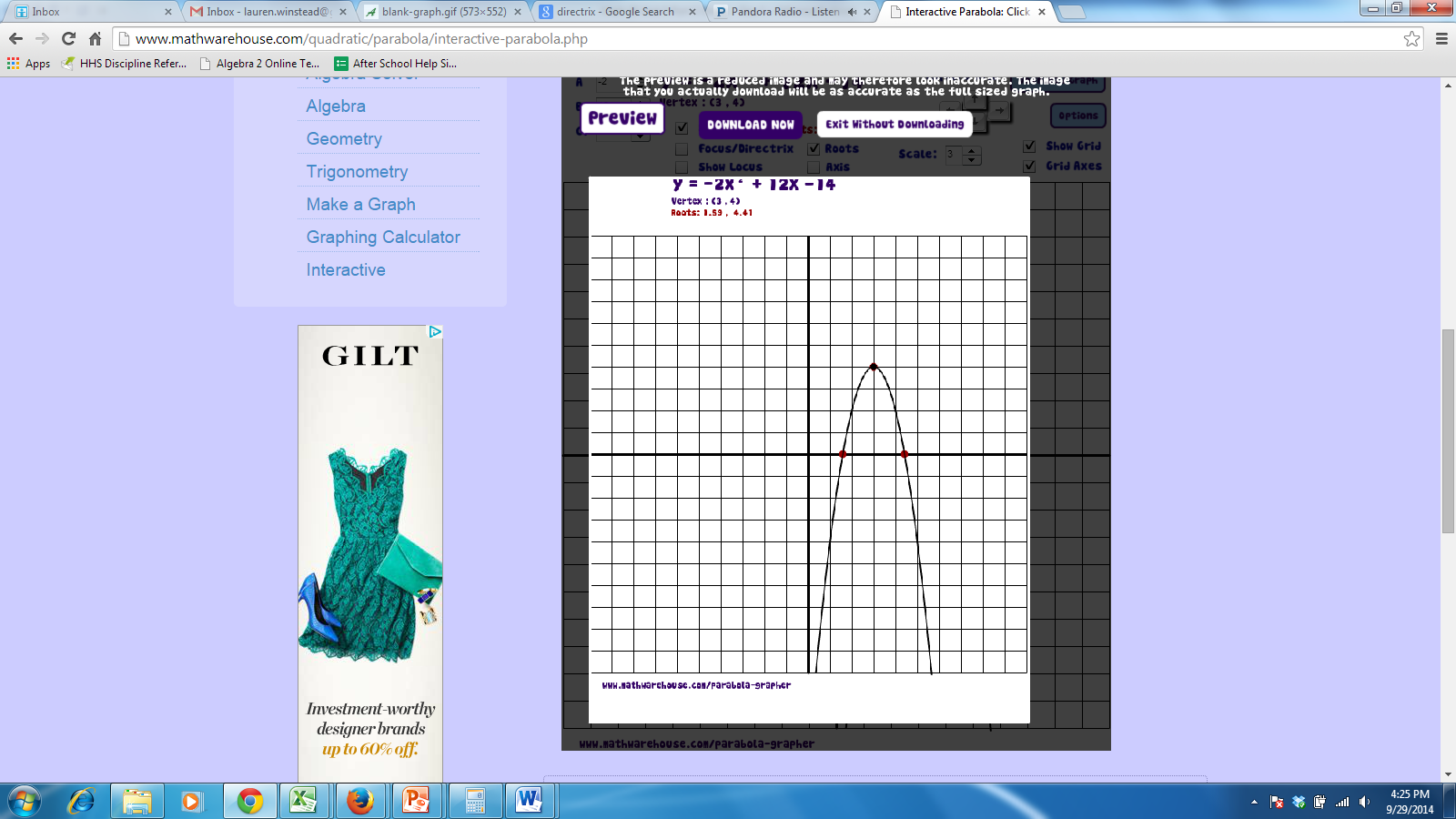
**Example 1:** Graph .

Where is the vertex? \_\_\_\_\_\_\_\_\_\_\_



**You try!** Graph

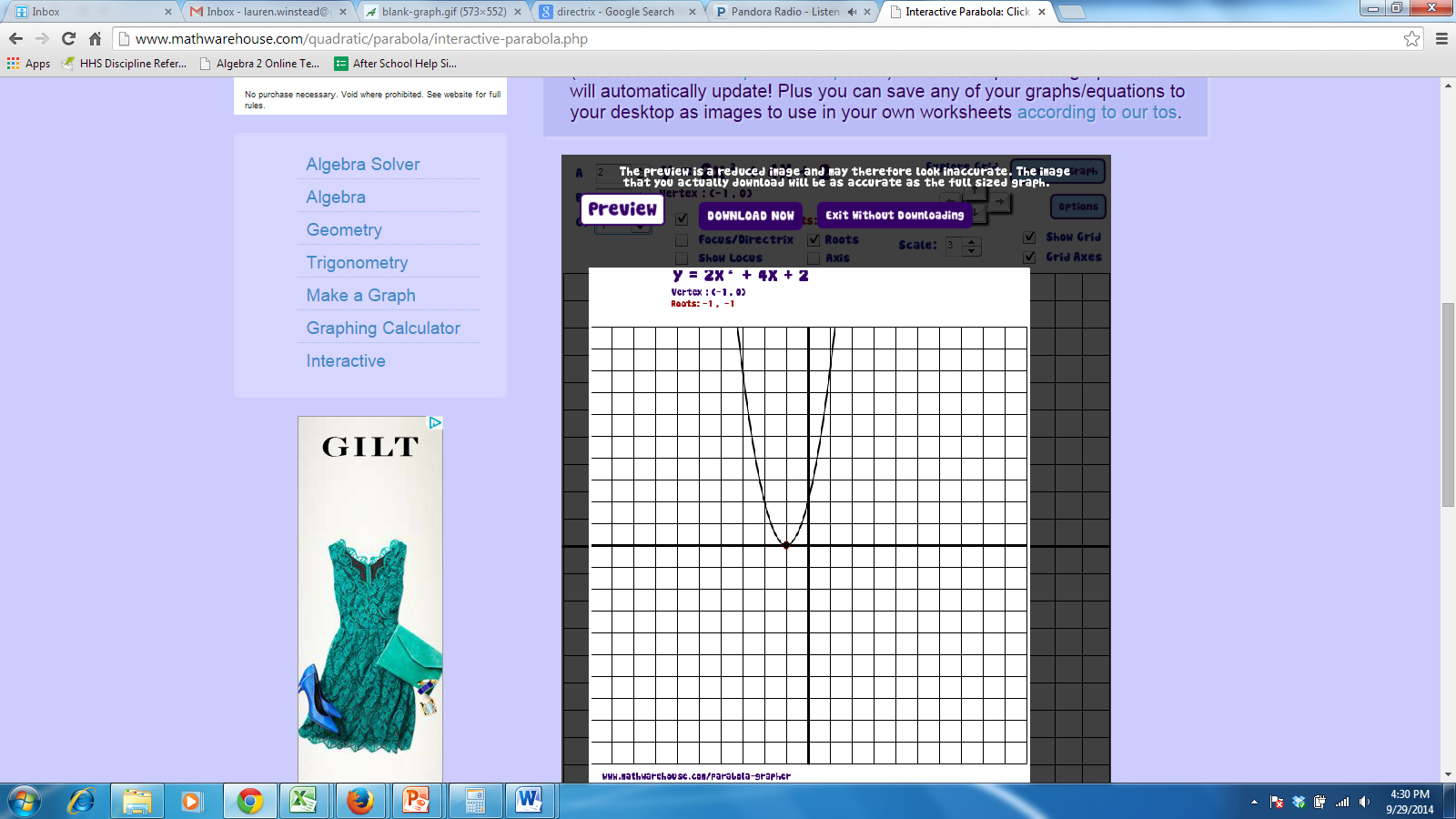
Where is the vertex? \_\_\_\_\_\_\_\_\_\_\_

**Writing the Equation of a Parabola in Vertex Form**

**Example 2**: Write the equation of the parabola given the graph.

**Step 1**: Plug the vertex into vertex form.

**Step 2**: Use one other point to solve for *a*.

**You try!** Write the equation of the parabola given the graph.

**Converting from Standard Form to Vertex Form**

**Example 3:** Convert to vertex form.

**Step 1:** Find the vertex.

**Step 2:** Plug the vertex into vertex form and pull *a* from the standard form equation.

**You try!** Convert to vertex form.

**Critical Thinking:**  How would you convert from vertex form back to standard form?

**Identifying Translations of Parabolas from Vertex Form**

**Rules for Transformations:**

***In***side the parentheses 🡪 ***In***verse of what you think

Negative # 🡪

Positive # 🡪

***O***utside the parentheses 🡪 ***O***bvious movement

Negative # 🡪

Positive # 🡪

Negative coefficient 🡪

Integer coefficient 🡪

Coefficient (fraction) 🡪

Graph , then graph .

What is different about the two graphs?

**Day 4: Focus and Directrix**

A parabola has two more important features known as the focus and the directrix.

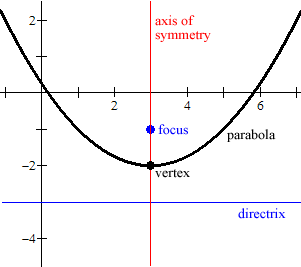
**Focus:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**Directrix:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

The distance between the vertex and the focus is called the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.



**Example 1:** Find the equation of the parabola with vertex at the origin and focus (0, 2).

Draw a picture first!



**Example 2:** What are the focus and directrix of the parabola with equation ?

**You try!** a) What is the equation of a parabola with vertex at (0, 0) and focus (0, -1.5)?

b) What are the vertex, focus, and directrix of the parabola with equation ?

**Example 3:** What are the vertex, focus, and directrix of the parabola with equation ?

First, get the equation in \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_!

**You try!** a)What are the vertex, focus, and directrix of the parabola with equation

?

1. What are the vertex, focus, and directrix of the parabola with equation ?

**Day 5: Completing the Square & Quadratic Formula**

Completing the Square is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

**Example 1:** Solve by completing the square.

**Step 1:** Move the constant to the other side.

**Step 2:** Compute and add the result to both sides of the equation.

**Step 3:** Convert the left side to a binomial squared and simplify the right side.

**Step 4:** Square root both sides, and don’t forget the on the right side!

**Step 5:** Solve for x. Remember that the gives you two solutions!

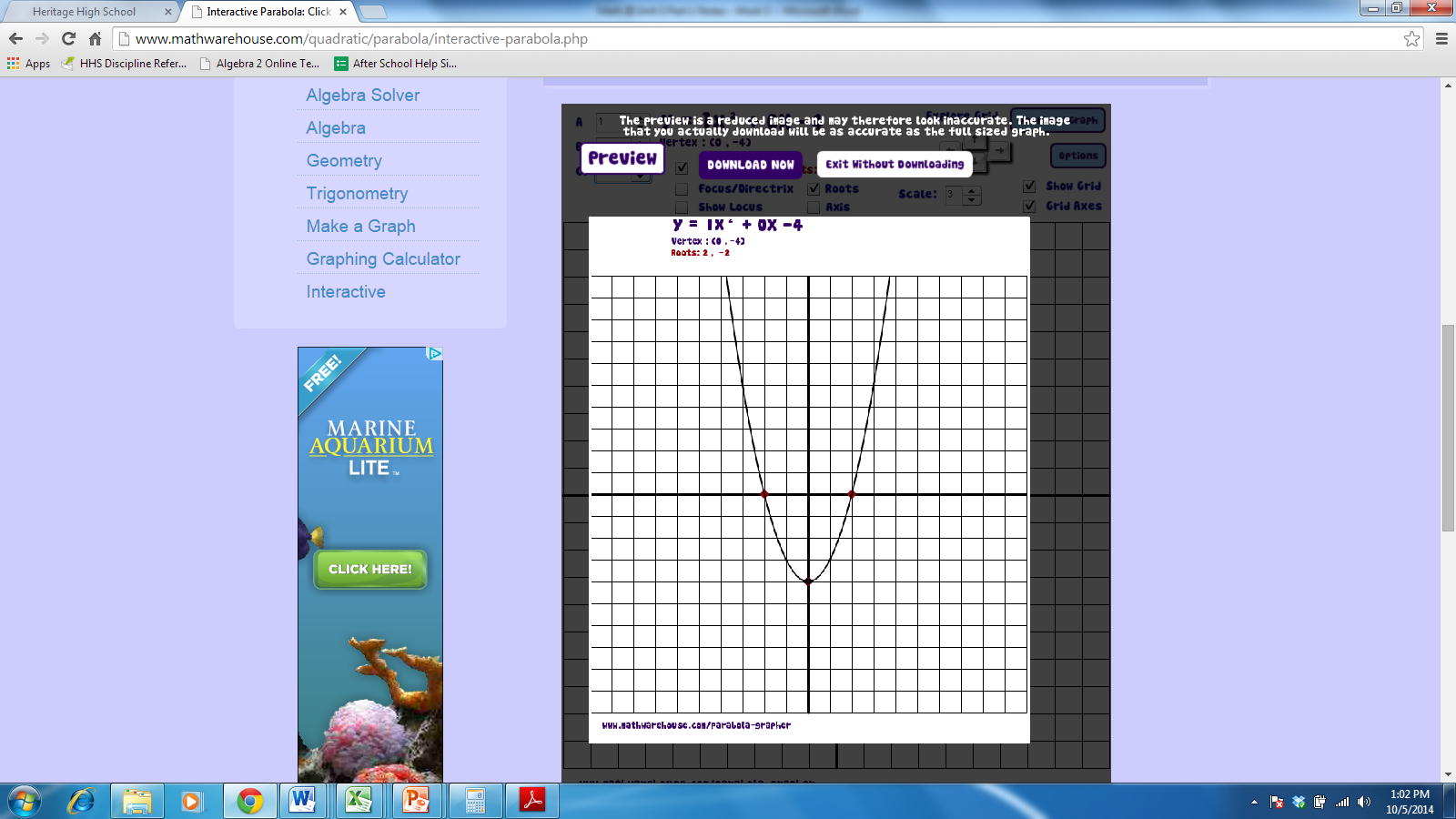
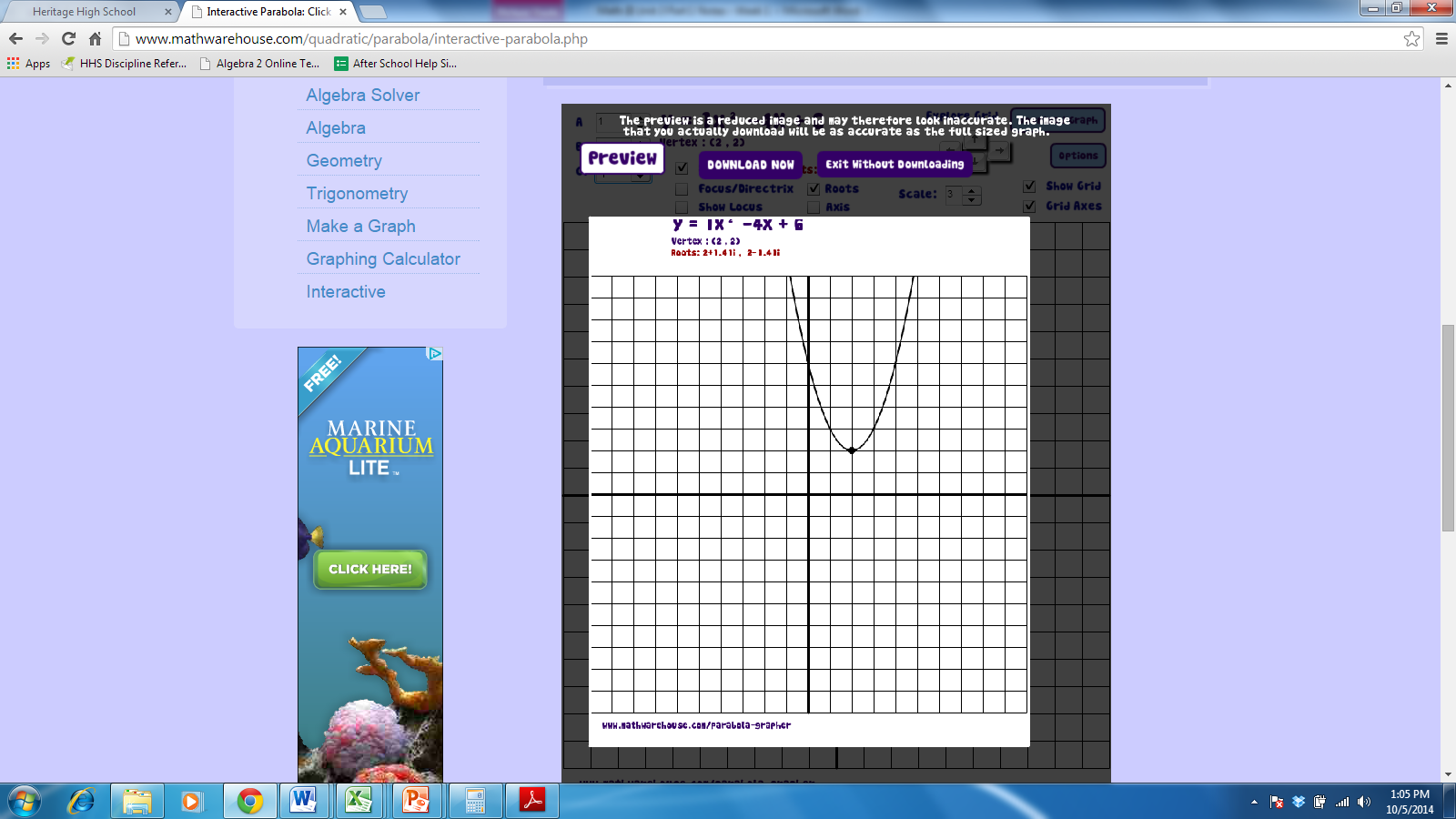
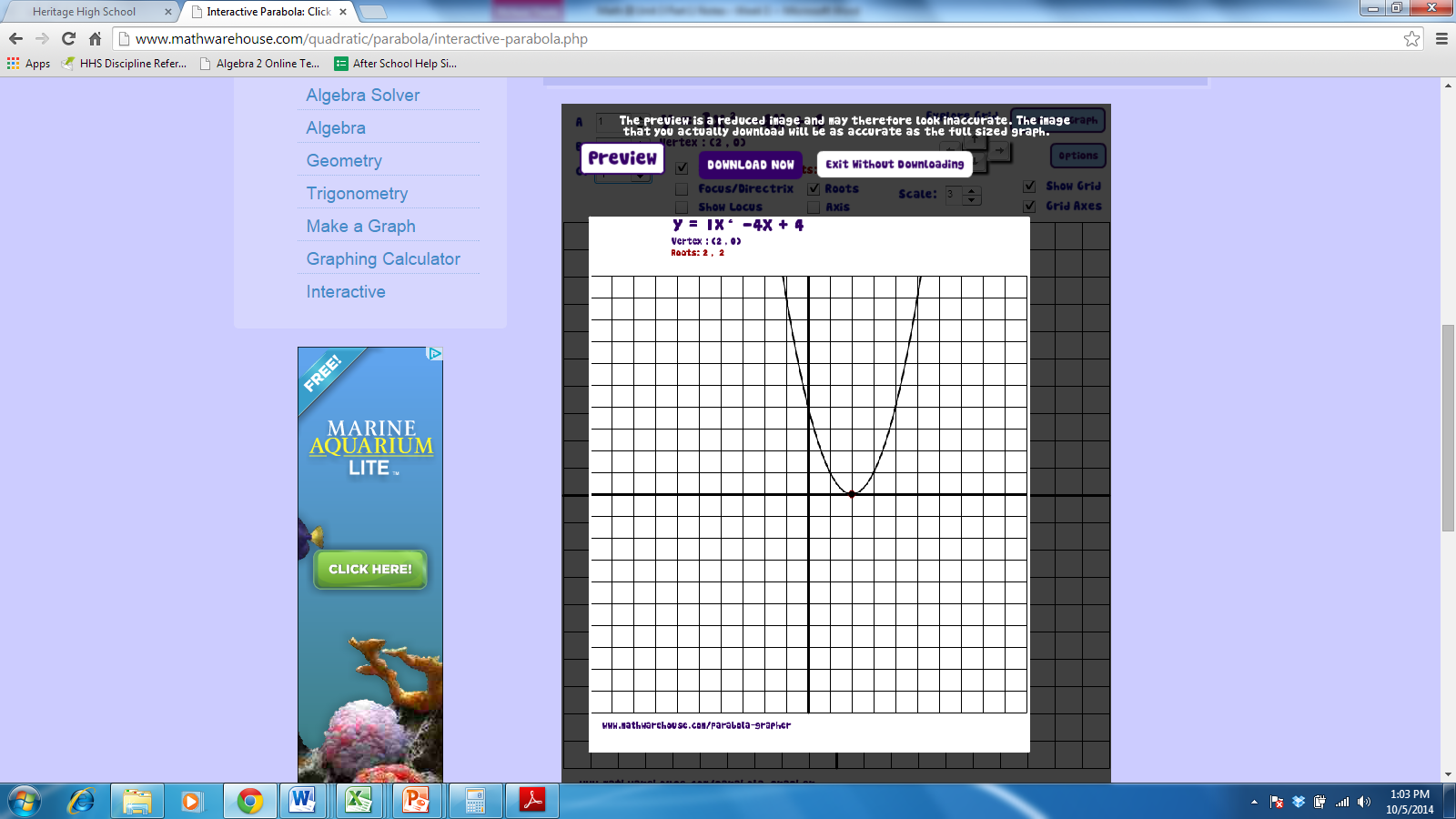
**Example 2:**

1. Solve by completing the square.

**You try!** Solve the following by completing the square. (It’s okay to get decimals!)

1. b)

**The Quadratic Formula**



How many solutions? \_\_\_\_\_ How many solutions? \_\_\_\_\_ How many solutions? \_\_\_\_\_

Type: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Type: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Type: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Discriminant is: \_\_\_\_\_\_\_\_\_\_ Discriminant is: \_\_\_\_\_\_\_\_\_\_ Discriminant is: \_\_\_\_\_\_\_\_\_\_

The quadratic formula is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What is the quadratic formula? Circle the discriminant!

**Example 1:** Use the discriminant to find the number and types of solutions to the quadratic expression. Remember to get all terms on one side and in standard form!

a) 3x2 – 5x - 18 b) 4x2 + 5 = 2x c) 2x2 = 3x – 12

**Example 2:** Use the quadratic formula to solve . Then, sketch the graph using what you know about the vertex and parabolic transformations.

**You try!** Use the quadratic formula to solve . Then, sketch the graph using what you know about the vertex and parabolic transformations.

**You try!** Use the quadratic formula to solve . Then, sketch the graph using what you know about the vertex and parabolic transformations.

**Modeling Using Quadratics**

2) The equation describes the height h, in feet, of a ball that is thrown straight up as a function of the time t, in seconds, that the ball has been in the air.

At what time does the ball reach its maximum height?

What is the maximum height?

When does the ball hit the ground?

1) Each year, Heritage’s Homecoming committee organizes a dance. Based on previous years, the organizers decided that the Income from ticket sales, I(t) is related to ticket price *t* by the equation *I(t) = 400t – 40t2.*

What ticket price(s) would generate the greatest income? What is the greatest income possible?

Ticket price(s) \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Income \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Day 6: Complex Number Operations**

You already know about real numbers (rational and irrational), but there are also \_\_\_\_\_\_\_\_\_\_\_\_\_\_ numbers that use the letter \_\_\_\_.

**Simplifying Using**

**Example 1:** Simplify .

**You try!** Simplify the following:a) b)

**Simplifying Complex Numbers**

Standard Form of a Complex Number:

**Example 2:** Write in standard form.

**You try!** Write in standard form.

**Adding and Subtracting**

**Example 3:** Simplify

**You try!** a) Simplify b) Simplify

**Multiplying Complex Numbers**

**Example 4:** Simplify .

**You try!** Simplify .

**Example 5:** Simplify . F:

O:

I:

L:

**You try!** a)Simplify . b) Simplify . c) Simplify .

**Rationalizing**

There is one big rule for complex number, and that is that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Why do you think this is? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Why do you think we call it rationalizing? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Rationalizing with One Term in the Denominator**

**Example 6:** Simplify **You try!** Simplify

What is a **complex conjugate?**

Why do we use it?

**Rationalizing with a Binomial in the Denominator**

**Example 7:** Simplify **You try!** Simplify

**Day 7: Finding Complex/Imaginary Solutions**

Quick review! Sketch the type of parabola that would have complex/imaginary roots.



Why does this parabola have imaginary roots?

Let’s solve some quadratic equations that have complex solutions!

**Example 1:** Solve .

**You try!** a) Simplify b) Simplify

**Example 2:** Solve **You try!** Solve

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Quadratic Equation | Value of Discriminant  (show work!) | Number of Solutions (or roots) | Types of Solutions (or roots) | Using the quadratic formula, what are the roots/solutions/zeros?  (show work!) |
|  |  |  |  |  |
|  |  |  |  |  |