Convert to logarithmic or exponential form:

Write each equation in exponential form.

13. $\log_2 16 = 4$

14. $\log_7 343 = 3$

15. $\log_9 \frac{1}{81} = -2$

16. $\log_3 \frac{1}{27} = -3$

17. $\log_{12} 144 = 2$

18. $\log_9 1 = 0$

Write each equation in logarithmic form.

19. $9^{-1} = \frac{1}{9}$

20. $6^{-3} = \frac{1}{216}$

21. $2^8 = 256$

22. $4^6 = 4096$

23. $27^{\frac{2}{3}} = 9$

24. $25^{\frac{3}{2}} = 125$

Evaluate each logarithmic expression using Change of Base Formula {or mental math}:

Evaluate each expression.

25. $\log_3 \frac{1}{9}$

26. $\log_4 \frac{1}{64}$

27. $\log_8 512$

28. $\log_6 216$

29. $\log_{27} 3$

30. $\log_{32} 2$

31. $\log_9 3$

32. $\log_{121} 11$

33. $\log_{\frac{1}{5}} 3125$

34. $\log_{\frac{1}{8}} 512$

35. $\log_{\frac{1}{3}} \frac{1}{81}$

36. $\log_{\frac{1}{6}} \frac{1}{216}$

Solving basic logarithmic equations:

1. $\log_4(7 - 8x) = 5$

2. $\log_5(4x - 1) = 3$

3. $\log_3(x - 17) = 3$

4. $\log_8(4 + 5x) = 2$

8. $\log_{81} x = \frac{3}{4}$

9. $\log_{25} x = \frac{5}{2}$

10. $\log_8 \frac{1}{2} = x$

11. $\log_6 \frac{1}{36} = x$

12. $\log_x 32 = \frac{5}{2}$

13. $\log_x 27 = \frac{3}{2}$

Solving exponential equations by finding common bases:

1. $3^{5x} = 27^{2x-4}$

2. $16^{2y-3} = 4^{y+1}$

3. $2^{6x} = 32^{x-2}$

4. $49^{x+5} = 7^{8x-6}$

Solving exponential equations using common logarithms:

23. $8^x = 40$

41. $3^x = 40$

42. $5^{3p} = 15$

25. $2.9^{a-4} = 8.1$

43. $4^{n+2} = 14.5$

44. $8^{z-4} = 6.3$

27. $13^{x^2} = 33.3$

45. $7.4^{n-3} = 32.5$

46. $3.1^{y-5} = 9.2$

Solving exponential equations using natural logarithms or base e:

34. $6e^x - 3 = 35$

35. $4e^x + 2 = 180$

36. $3e^{2x} - 5 = -4$

37. $-2e^{3x} + 19 = 3$

38. $6e^{4x} + 7 = 4$

39. $-4e^{-x} + 9 = 2$

5A. $5 \ln 6x = 8$

25. $\ln (x + 4) = 36$

. $\ln 2x = 6$

KeyConcept Continuosly Compounded Interest

Calculate continuously compounded interest using the following formula.

$$A = Pe^{rt},$$

where A is the amount in the account after t years, P is the principal amount invested, and r is the annual interest rate.

KeyConcept Compound Interest

You can calculate compound interest using the following formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt},$$

where A is the amount in the account after t years, P is the principal amount invested, r is the annual interest rate, and n is the number of compounding periods each year.

You can model exponential growth with a constant percent increase over specific time periods using the following function.

$$A(t) = a(1 + r)^t$$

Similar to exponential growth, you can model exponential decay with a constant percent of decrease over specific time periods using the following function.

$$A(t) = a(1 - r)^t$$

An investment account pays 4.2% annual interest compounded monthly. If \$2500 is invested in this account, what will be the balance after 15 years?

. Find the balance of an account after 7 years if \$700 is deposited into an account paying 4.3% interest compounded monthly.

. Determine how much is in a retirement account after 20 years if \$5000 was invested at 6.05% interest compounded weekly.

. A savings account offers 0.7% interest compounded bimonthly. If \$110 is deposited in this account, what will the balance be after 15 years?

. A college savings account pays 13.2% annual interest compounded semiannually. What is the balance of an account after 12 years if \$21,000 was initially deposited?

FINANCIAL LITERACY When Angelina was born, her grandparents deposited \$3000 into a college savings account paying 4% interest compounded continuously.

a. Assuming there are no deposits or withdrawals from the account, what will the balance be after 10 years?

If they invested \$8000 at 3.75% interest compounded continuously, how much money would be in the account in 30 years?

If they could only deposit \$10,000 in the account above, at what rate would the account need to grow in order for Angelina to have \$30,000 in 18 years?

If Angelina's grandparents found an account that paid 5% compounded continuously and wanted her to have \$30,000 after 18 years, how much would they need to deposit?

. **FINANCIAL LITERACY** The value of a certain car depreciates according to $v(t) = 18500e^{-0.186t}$, where t is the number of years after the car is purchased new.

a. What will the car be worth in 18 months?

b. When will the car be worth half of its original value?

c. When will the car be worth less than \$1000?

CARS Abe bought a used car for \$2500. It is expected to depreciate at a rate of 25% per year. What will be the value of the car in 3 years?

BIOLOGY For a certain strain of bacteria, k is 0.728 when t is measured in days. Using the formula $y = ae^{kt}$, how long will it take 10 bacteria to increase to 675 bacteria?

POPULATION The population of a city 20 years ago was 24,330. Since then, the population has increased at a steady rate each year. If the population is currently 55,250, find the annual rate of growth for this city.

SAVINGS You deposited \$1000 into an account that pays an annual interest rate r of 5% compounded quarterly. Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

a. How long will it take until you have \$1500 in your account?

b. How long it will take for your money to double?

Classwork/Homework:

) **FINANCIAL LITERACY** Use the formula for continuously compounded interest.

a. If you deposited \$800 in an account paying 4.5% interest compounded continuously, how much money would be in the account in 5 years?

b. How long would it take you to double your money?

c. If you want to double your money in 9 years, what rate would you need?

d. If you want to open an account that pays 4.75% interest compounded continuously and have \$10,000 in the account 12 years after your deposit, how much would you need to deposit?

) **BIOLOGY** A certain bacteria is growing exponentially according to the model $y = 80e^{kt}$, where t is the time in minutes.

a. If there are 80 cells initially and 675 cells after 30 minutes, find the value of k for the bacteria.

b. When will the bacteria reach a population of 6000 cells?

A certain culture of bacteria will grow from 250 to 2000 bacteria in 1.5 hours. Find the constant k for the growth formula. Use $y = ae^{kt}$.

POPULATION The population of a city 10 years ago was 150,000. Since then, the population has increased at a steady rate each year. The population is currently 185,000.

a. Write an exponential function that could be used to model the population after x years if the population changes at the same rate.

b. What will the population be in 25 years?

i. **AGRICULTURE** An equation that models the decline in the number of U.S. farms is $y = 3{,}962{,}520(0.98)^x$, where x is the number of years since 1960 and y is the number of farms.

a. How can you tell that the number is declining?

b. By what annual rate is the number declining?

c. Predict when the number of farms will be less than 1 million.

SAVINGS You put \$7500 in a savings account paying 3% interest compounded continuously.

a. Assuming there are no deposits or withdrawals from the account, what is the balance after 5 years?

b. How long will it take your savings to double?

c. In how many years will you have \$10,000 in your account?

Jason recently purchased a new truck for \$34,750. The value of the truck decreases by 12% each year. What will the approximate value of the truck be 7 years after Jason purchased it?

A \$13,775

B \$13,890

C \$14,125

D \$14,200

KeyConcept Product Property of Logarithms

Words	The logarithm of a product is the sum of the logarithms of its factors.
Symbols	For all positive numbers a, b, and x, where $x \neq 1$, $\log_x ab = \log_x a + \log_x b$.
Example	$\log_2 [(5)(6)] = \log_2 5 + \log_2 6$

KeyConcept Quotient Property of Logarithms

Words	The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.
Symbols	For all positive numbers a, b, and x, where $x \neq 1$, $\log_x \frac{a}{b} = \log_x a - \log_x b$.
Example	$\log_2 \frac{5}{6} = \log_2 5 - \log_2 6$

KeyConcept Power Property of Logarithms

Words	The logarithm of a power is the product of the logarithm and the exponent.
Symbols	For any real number p, and positive numbers m and b, where $b \neq 1$, $\log_b m^p = p \log_b m$.
Example	$\log_2 6^5 = 5 \log_2 6$

Solve each equation. Check your solutions.

23. $\log_3 56 - \log_3 n = \log_3 7$

24. $\log_2 (4x) + \log_2 5 = \log_2 40$

25. $5 \log_2 x = \log_2 32$

26. $\log_{10} a + \log_{10} (a + 21) = 2$

Solve each equation. Check your solutions.

36. $\log_3 6 + \log_3 x = \log_3 12$

37. $\log_4 a + \log_4 8 = \log_4 24$

38. $\log_{10} 18 - \log_{10} 3x = \log_{10} 2$

39. $\log_7 100 - \log_7 (y + 5) = \log_7 10$

40. $\log_2 n = \frac{1}{3} \log_2 27 + \log_2 36$

41. $3 \log_{10} 8 - \frac{1}{2} \log_{10} 36 = \log_{10} x$

ANSWERS BELOW:

23.

24.

25.

26.

36.

37.

38.

39.

40.

41.