

Parent function of exponential functions  $f(x) = b^x$

$b > 1$ ; {exponential growth}

$0 < b < 1$ ; {exponential decay}

Recognizing transformations:  $f(x) = a \cdot b^{x \pm h} \pm k$

If there is an  $(-a) \cdot b$ , then the function is reflected over the  $x$  - axis and it is neither growth nor decay (but still an exponential function). If there is a  $-x$  (in the exponent), the function reflects over the  $y$  - axis.

$|a| > 1$ ; vertical stretch by a factor of  $a$

$0 < |a| < 1$ ; vertical compression by a factor of  $a$

$(x + h)$ ; shift left  $h$  units;  $(x - h)$ ; shift right  $h$  units

$+k$ ; shift up  $k$  units;  $-k$ ; shift down  $k$  units

Asymptote: an imaginary line that the function gets really close to but never touches or crosses {hint: look @  $k$ } Write this as an equation:  $y = k$

$Y$  - intercept: when  $x = 0$

**Exponential Growth and Decay** Many real-world situations can be modeled by exponential functions. One of the equations below may apply.

| Exponential Growth or Decay  | Continuous Exponential Growth or Decay  | Compound Interest  |
|--|---|--|
| $N = N_0(1 + r)^t$   | $N = N_0e^{rt}$   | $A = P\left[1 + \frac{r}{n}\right]^{nt}$   |
| $N$ is the final amount, $N_0$ is the initial amount, $r$ is the rate of growth or decay, and $t$ is time. | $N$ is the final amount, $N_0$ is the initial amount, $r$ is the rate of growth or decay, $t$ is time, and $e$ is a constant. | $P$ is the principal or initial investment, $A$ is the final amount of the investment, $r$ is the annual interest rate, $n$ is the number of times interest is compounded each year, and $t$ is the number of years. |

**Example 1 BIOLOGY** A researcher estimates that the initial population of a colony of cells is 100. If the cells reproduce at a rate of 25% per week, what is the expected population of the colony in six weeks?

$$\begin{aligned} N &= N_0(1 + r)^t && \text{Exponential Growth Formula} \\ &= 100(1 + 0.25)^6 && N_0 = 100, r = 0.25, t = 6 \\ &\approx 381.4697266 && \text{Use a calculator.} \end{aligned}$$

There will be about 381 cells in the colony in 6 weeks.

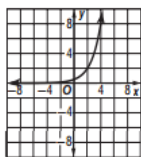
**Example 2 FINANCIAL LITERACY** Lance has a bank account that will allow him to invest \$1000 at a 5% interest rate compounded continuously. If there are no other deposits or withdrawals, what will Lance's account balance be after 10 years?

$$\begin{aligned} A &= Pe^{rt} && \text{Continuous Compound Interest Formula} \\ &= 1000e^{(0.05)(10)} && P = 1000, r = 0.05, \text{ and } t = 10 \\ &\approx 1648.72 && \text{Simplify.} \end{aligned}$$

With continuous compounding, Lance's account balance after 10 years will be \$1648.72.

Sketch and analyze the graph of each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

1.  $f(x) = 2^{x-1}$



$D = (-\infty, \infty);$

$R = (0, \infty);$

y-intercept:  $(0, \frac{1}{2});$

x-intercept: none;

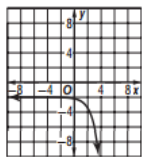
asymptote: x-axis;

end behavior:  $\lim_{x \rightarrow -\infty} f(x) = 0$

and  $\lim_{x \rightarrow \infty} f(x) = \infty;$

increasing:  $(-\infty, \infty)$

2.  $h(x) = -\frac{1}{5}e^x - 2$



$D = (-\infty, \infty);$

$R = (-\infty, -2);$

y-intercept:  $(0, -2.2);$

x-intercept: none;

asymptote:  $y = -2;$

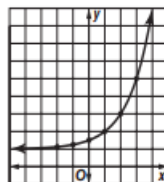
end behavior:  $\lim_{x \rightarrow -\infty} f(x) = -2$

and  $\lim_{x \rightarrow \infty} f(x) = -\infty;$

decreasing:  $(-\infty, \infty)$

Sketch and analyze the graph of each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

1.  $h(x) = 2^{x-1} + 1$



$D = (-\infty, \infty);$

$R = (1, \infty);$

intercept:  $(0, 1 \frac{1}{2});$

asymptotes:  $y = 1;$

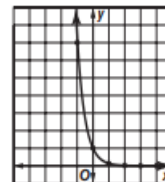
end behavior:

$\lim_{x \rightarrow -\infty} f(x) = 1$  and

$\lim_{x \rightarrow \infty} f(x) = \infty;$

increasing:  $(-\infty, \infty)$

2.  $h(x) = e^{-2x}$



$D = (-\infty, \infty);$

$R = (0, \infty);$

intercept:  $(0, 1);$

asymptote: x-axis;

end behavior:

$\lim_{x \rightarrow -\infty} f(x) = \infty$  and

$\lim_{x \rightarrow \infty} f(x) = 0;$

decreasing:  $(-\infty, \infty)$

Identify: domain, range, y-intercept, x-intercept, asymptote, end behavior, interval

increasing/decreasing, transformations from parent function:  $f(x) = (\frac{4}{3})^{x+5} - 10$

Identify: domain, range, y-intercept, x-intercept, asymptote, end behavior, interval

increasing/decreasing, transformations from parent function:  $f(x) = (0.75)^{x-2} + 3$